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Casualty Reinsurance Exposure rating - Moving into the stochastic realm

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Abstract

When pricing casualty reinsurance, the actuary often faces the problem of experience data that have only very limited credibility. Exposure rating using Increased Limit Factors (ILFs) can offer a useful complement to experience rating. In this paper we provide an introduction to the theory of Increased Limits Factors, review examples of common families of ILF curves, explain how ILFs can be used in excess insurance rating and reinsurance rating, and offer some guidance on setting up simulations using ILF severities.

Key words

Increased Limits Factors (ILFs); Liability; Pricing; Stochastic; Reinsurance; Risk Profile

1. Introduction

Pricing casualty reinsurance can be difficult; parameterisation can be very tricky and judgemental. This can be compounded by limited number of claims and, claims from policies with limits. Fitting a loss distribution is fraught with many problems, not least of which is how to adjust for inflation and development. Also there may be an absence of large claims in the loss history, and absence of evidence should not be taken as evidence of absence, i.e. "just because it hasn't happened yet doesn't mean that it can't or it won't". Or there may be too many large losses. This can lead to erratic results and uncertainty in pricing excess layered business if using frequency and severity or experienced based methods. Things can become more complex when policies with a variety of limits and deductibles are used and where the risk assumed is not 100% (co –insurance).

Often in liability reinsurance there is a limited amount of claims data available, and due to the statistical variation in large losses there will always be a limit to how credible pricing estimates can be using experience based methods. Increased limits factors can provide stability in pricing estimates. A historical record of consistent technical prices can become a valuable source of sustainable competitive advantage; with additional information gained with respect to the uncertainty surrounding these estimates can prove useful additional information to enable better strategic planning allowing management and underwriters to better manage the insurance cycle.

The usual method of exposure rating property using standard well-known inflation-independent exposure rating curves is not valid in liability insurance in the same way because here the claims sizes cannot be assumed to be scaled by the sums insured. Thus we need another method to apply to a risk profile to deduce expected losses above various thresholds. This is especially vital when claims data is sparse and the insights gained can be useful in comparing claims experience to date with that expected. Exposure rating allows us to take on objective view of whether there have been too many or too few large losses by having a measure that is not based upon the data itself (as a loss severity curve would be). We must also not lose sight of the fact that when exposure rating in general, the underlying loss ratio of the insurance business is a major factor influencing any reinsurance pricing.

2 Limitations

Before addressing the critical elements of our technical analysis, we should pause to give a moment's consideration to the inevitable real-world limitations of some of the theory here.

We might take the example of the legal world of professional indemnity for solicitors. Here there are many small firms handling small scale real estate transfers and small scale domestic disputes and litigation. However there are also a small number of very large firms, the biggest among which are leading the field in immense merger and acquisition transactions. This second type of activity is largely unknown to the smaller and middle-sized firms. For the small firms, the ILF's have been analysed and have been shown to be remarkably low - the price of \$1m xs 1m for many small and middle-sized legal firms is often under 10% of the cost of the primary \$1m. However for very large firms, the risk of \$10m xs 90m is not much reduced from the risk of \$10m xs 50m, which implies a dramatically higher ILF.

Comparable problems arise in many fields. The ability of statisticians to roll data together across an occupation or a profession whilst maintaining legitimacy, is often going to be hampered by this kind of situation. So whilst the following work hopefully gives rise to a deeper understanding of the issues in developing this type of analytic capability, the work is always rooted in a core assumption that the underlying data has sufficient basic comparabilities. This assumption always needs to be tested in the actual-world reality before firm conclusions can be drawn.

3. Increased Limit Factors

In casualty insurance a loss is typically covered up to a specified limit. Assuming that the expected loss of a book is known when the limit is at a basic level, this expected loss clearly increases when the limit is raised. The ratio by which it increases is called the Increased Limit Factor. We'll explore the distribution of the loss when the Increased Limit Factor is given for all limits above the basic level.

We'll formalise the definition in a moment but first note that for stochastic modelling the ILF curve must be translated into a loss model. We'll work with the standard insurance loss model which is described by the number of losses in a period (typically a year) described by the random variable N and a sequence of independent identically distributed losses X_1, X_2, X_3, K all assumed to have the same distribution as a random variable X. The sequence of the losses is also assumed to be independent from the number of losses. The total loss S in a year is then given as $S = \sum_{k=0}^{N} X_k$, and it is a well-known fact that the total expected loss can be calculated

$$E(S) = E(N) \times E(X). \tag{3.1}$$

When ratemaking insurers generally cap large losses at a suitable limit (class dependant), which would skew the experience of a particular combination of rating factors or set of rates. That is large losses would appear as a full limit loss. The rates then produced would be for a set limit. In order to sell policies for limits greater than this limit, an extra premium needs to be charged. The calculation of this premium would be done by multiplying the base premium by an *Increased Limit Factor(ILF)*.

An alternative definition of an ILF, based on a size of loss distribution, is the ratio of the limited expected values. The formula is shown below:-

Increased Limit Factor
$$(y) = \frac{E[f(x;y)] \times E(n)}{E[f(x;L)] \times E(n)} = \frac{E[f(x;y)]}{E[f(x;L)]} = \frac{E[\min(X,y)]}{E[\min(X,L)]}$$
 (3.2)

Where
$$E[f(x; y)] = \int_{0}^{y} xf(x)dx + y[1 - F(y)]$$
 (3.3)

I.e. the limited expected value

x =Random variable

f(x) = probability density f unction of x (the severity distribution)

F(x) = cumulative distribution function of x

E(n) = the expected number of losses (from ground up)

y = limit required

L = Basic Limit

In general we can refer to the L as the Basic limit and it can be interpreted in pricing terms as the point where the large losses would be capped for the purpose of calculating your premium relativities to avoid getting distortions from large claims. Clearly this L does depend to some extend on the volume of data that you may have in estimating your original pricing rates, as with sparse data then almost any claim you could argue is a distortion.

As can be seen the ILFs are based purely on severity distributions (the frequency terms cancel out).

When calculating Increased Limit factors the data may be sparse at high loss severities so a loss distribution will be used to extrapolate the higher factors. Also there is the difficulty for liability lines of claims inflation and development, where in practice inflation only may be used due to the sparseness of the data at higher loss levels. Also it should be recognised that judgement is required in setting ILF factors of the underlying severity distribution, in many cases judgement on the factors themselves may be used. We mention later some useful simple checks that can be used to ensure that any judgemental factors are themselves consistent and do not leave room for antiselection.

We first recap some of the basic ILF formulae found in Miccolis (1977). These are derived below for convenience, and for a fuller discussion we refer the reader to Miccolis.

The definition of the ILF formula is shown below:-

$$ILF(y) = \frac{E[(\min(X, y)]]}{E[(\min(X, L)]]} = \frac{E[(\min(X, y))]}{B}$$
Where B =E[\min(X, L)] is the expected loss limited to basic limits
$$= \frac{1}{B} \times \left(\int_{0}^{y} xf(x)dx + y[1 - F(y)] \right)$$
So that

$$\frac{dILF(y)}{dy} = ILF'(y) = \frac{1}{B} \times \left(y \frac{dF(y)}{dy} + 1 - F(y) - y \times \frac{dF(y)}{dy} \right) \quad \text{Note f(y)} = F'(y) = dF(y)/dy$$

Thus

$$ILF'(y) = \frac{1 - F(y)}{B}$$
 (3.4)

We can restate this equation as

$$1 - P\{Y \le y\} = 1 - F(y) = E[(\min(X, L)] \times ILF'(y)]$$
(3.5)

This is significant as it shows the relationship between the underlying severity distribution and the ILF curve. In words, it tells us that, given an ILF curve above the basic limit L, we know the severity distribution of the losses above L. No information is given below the threshold which we obviously cannot expect. This observation was further analysed in Mack (2003) and we refer the interested reader to their article.

The interpretation of this formula leads to some useful checks that can be applied to ILF factors. As $y \to \infty$ then $F(y) \to 1$ so that $ILF'(y) \to 0$, the interpretation of this is that the ILF factors converge to a constant value, for some value. This implies that there is no additional charge for limits above this level (strictly speaking, the additional charge tends to zero). In a practical sense this can be a very high limit where we can see no possibility of a loss this size, and would be the largest limit on offer, where this point lies may have commercial implications. Commercially what happens is that you glide imperceptibly, from a layer that's recognisably priced in relation to the risk of loss, into a layer that's priced only in relation to the cost of the capital deployed.

Also since
$$f(y) = \frac{dF(y)}{dc} = -E(\min(Y,b) \times ILF''(y))$$
 so that $ILF''(y) = \frac{-f(y)}{E(\min(Y,b))}$

$$(3.6)$$

That is the 2^{nd} derivative of the ILF function is always <=0 (as f(y) is >=0, and E(min(Y,b))>0), so that the ILF'(y) must be monotonically decreasing (i.e. the 2^{nd} derivative cannot be positive; that is, the additional charge per unit of limit at a higher attachment cannot be greater than the charge at a lower attachment), and ILF(y) must be strictly increasing (can be stationary, as shown above).

These properties are very important properties of ILFs and are worth summarising and restating:-

- (i) ILF values are non decreasing.Any scale of ILFs should always increase.
- (ii) ILF values are asymptotically constant.

 Any scale of ILFs should always approach a constant value.
- (iii) ILF curves are concave down.

 Any scale of ILFs should increase at a decreasing rate.

This leads us to a simple practical tests to estimate whether ILFs that have emerged over time (i.e. through judgment and continual massaging), are consistent with these rules. The ILF must always increase (they can level off at a suitably high level of loss), and they must increase at a decreasing rate, so that we can test the differences between the ILFs. If these rules are not met, then there exists the possibility to get

pricing inconsistencies, which no doubt would be discovered by the market and lead to anti-selection.

An example may help to visualise this, below we show a set of ILFs that violate the ILF properties:-

Limit	ILF	First Order	Second Order
		Difference	Differences
\$1,000,000	1.000	2.00E-7	+0.00E0
\$2,000,000	1.200	2.00E-7	-1.50E-7
\$3,000,000	1.400	5.00E-8	+2.70E-8
\$4,000,000	1.450	7.70E-8	-2.03E-9
\$5,000,000	1.527	6.63E-8	
\$6,000,000	1.602		

Here we see that the probability of a loss exceeding \$1m is the same as the probability of exceeding \$2m, so that there is no probability of a loss in the range \$1m to \$2m. We also see that the probability of getting a loss above \$4m is greater than the probability of exceeding \$3m, thus there is a negative probability of being in the range \$3m to \$4m. These ILF may look sensible when you cast your eye down their absolute values, but with a little work we can see that they will not generate sensible or consistent prices.

Given a set of ILFs it is possible to reverse engineer the severity distribution that was used to create them. This can be useful as it may be easier to explain and validate this distribution than to discuss the ILFs. It may also be easier to discuss the probability of various loss sizes, than ILF factors directly.

4. Parametric Formulations of ILFs

4.1 Riebesell

This is based upon the work of the German Actuary Paul Riebesell (1936). Suppose the standard risk premium for a limit of L is $P_L = P(L)$. Then, if the limit (or sum insured) is for example doubled, the amount of risk premium required will be $P(2L) = P_L(1+z)$, i.e., an increase by a factor of (1+z). The general rule can be easily derived as $P(2^i L) = P_L(1+z)^i$. By using a substitution of $t = 2^i$, i.e., $i = \log(t,2)$ (i.e., log base 2) this can be rewritten as $P(tL) = P_L(1+z)^{\log(t,2)}$. This formula can be written more helpful to give the premium for any desired limit in terms of the relativity to the

base premium (i.e. substitute L = tL, $P(y) = P_L(1+z)^{\log\left(\frac{y}{L},2\right)}$). By using the law of logs, this can be written most conveniently as $P(y) = P_L\left[\left(\frac{y}{L}\right)^{\log(1+z,2)}\right]$, from this

$$ILF_R(y) = \left(\frac{y}{L}\right)^{\log(1+z,2)}$$
(4.1)

we can define the ILF such that:-

So that $P(y) = P_L$. $ILF_R(y)$, and we can see how easy the ILFs are to apply in practice.

4.1.1 Some Properties of z

- $z \in (0,1)$
- z is greater than 0. If a larger limit is charged, then there should be more premium charged. However, if this were not the case, then we would be assuming some limit on the losses such that the increase in limit generates no additional loss cost.
- z is less than 1. If 1 (or more) is selected, then we are assuming the increase in cost is linear. This does not apply to any claims distribution applied in practice, as larger losses have a smaller probability of occurring, indeed for any uncapped loss distribution, this is necessarily the case.

We could also generalise the formula in the sense that the factor of 2 may not be "exactly right". Many ILFs in practice can be well approximated using a natural logarithm curve for smoothing, suggesting a factor of Euler's number (e = 2.718281828...). From a pragmatic perspective, the factor of 2 appears to be reasonable in a wide range of circumstances.

Mack & Fackler (2003) demonstrated that there exist loss distributions that lead to Riebesell's rule, and it is not intended to cover this ground again here.

4.2 Pareto

An equivalent version (a simple re-parameterisation) is the Pareto curve

$$ILF_{P}(x) = \left(\frac{x}{b}\right)^{(1-\beta)} \tag{4.2}$$

Whose name suggests a connection with a Pareto severity distribution. This was indeed demonstrated in MF.

The advantages of the Pareto model:-

- The simplicity of the model (i.e. a single parameter from a well understood loss distribution).
- It is possible to get a feel for the parameters for particular classes, consistent with studies of loss data. For example, liability parameters Pareto parameter 0.7 to 1, property parameters 1-1.9.
- It can be used with loss experience to rate using a mixture of exposure (the Pareto model) and experience (using the actual losses to estimate the parameter)
- Simple to express rates as mathematical formulas

Disadvantages

- Small judgemental changes in parameter values can generate large swings in resultant prices
- Has a heavy tail
- Is over simplistic, and could be abused to justify a price which may not be adequate

4.3 Mixed Exponential

This leads us to the question if other parametric ILF curves can be characterised. For example, a severity distribution of exponential type, which has cumulative distribution function $F(x)=1-\exp\{-x/\lambda\}$ leads to an ILF:-

$$ILF_E(x) = \frac{1 - \exp\{-x/\lambda\}}{1 - \exp\{-L/\lambda\}}$$
 (4.3)

This can be seen by first calculating $E[\min(X,L)] = \lambda(1 - \exp\{-L/\lambda\})$ and then, applying equation 1, it follows

$$ILF'_{E}(x) = \frac{P\{X > x\}}{E[\min(X, L)]}$$

$$= \frac{\exp\{-x/\lambda\}}{\lambda(1 - \exp\{-L/\lambda\})}$$
(4.4)

and the result follows by integration.

As mentioned earlier, there are a large number of mixed exponential curves (Keatinge (1999) that are used in casualty modelling. The Insurance Services Office "ISO" have written extensively on ILF issues (White S & Mrazek K 2004, Thorpe (2008), Svenguard (2004).

We need to formalise the notion of a mixed distribution before we derive their ILF_E.

Given a number of n independent random variables Y_1, Y_2, K , Y_n with weights w_1, w_2, K , w_n (the weights must sum to 1). They can be "mixed" in the following way: first we pick the random variable (i.e. we pick a number $k, 1 \le k \le n$), then we sample from the random variable Y_k . This heuristic idea of mixing random variables can be mathematically formalised by a discrete random variable K with outcomes K, $1 \le k \le n$, each with probability W_k . The mixed distribution is then represented by the random variable Y_K which has cumulative distribution function $F(x) = \sum_{k=1}^n w_k F_k(x)$ which has, for the exponential case, the form

$$F_{ISO}(x) = \sum_{k=1}^{n} w_k \left(1 - \exp\{-x/\lambda_k\} \right).$$
 (4.5)

As above, we start with the calculation of the limited mean $E[\min(X_K, L)] = \sum_{k=1}^{n} w_k E[\min(X_k, L)]$

$$\sum\nolimits_{k=1}^{n} w_k \lambda_k \left(1 - \exp\left\{-L/\lambda_k\right\}\right)$$

The ILF curve is then

$$ILF'_{ISO}(x) = \frac{P\{X_K > x\}}{E[\min(X_K, L)]}$$

$$= \frac{1 - \sum_{k=1}^{n} w_k (1 - \exp\{-x/\lambda_k\})}{E[\min(X_K, L)]}$$

$$= \frac{\sum_{k=1}^{n} w_k \exp\{-x/\lambda_k\}}{E[\min(X_K, L)]}$$
(4.6)

and integration yields the result

$$ILF_{ISO}(x) = \frac{\sum_{k=1}^{n} w_k \lambda_k (1 - \exp\{-x/\lambda_k\})}{E[\min(X_K, L)]}$$

$$= \frac{\sum_{k=1}^{n} w_k \lambda_k (1 - \exp\{-x/\lambda_k\})}{\sum_{k=1}^{n} w_k \lambda_k (1 - \exp\{-L/\lambda_k\})}$$
(4.7)

The ISO office provides the parameters underlying the mixed exponential, and by using suitable adjustments they can be adjusted for inflation. Since 1971, ISO has been a leading source of information about US risk; they supply data, analytics, and decision-support services for professionals in many fields.

5. Excess of Loss Reinsurance Rating

In order to calculate a risk premium for an excess of loss reinsurance layer, using ILFs we need the loss ratio of the underlying business. To convert this into an office premium we need some further assumptions regarding the loss ratio required by the excess writer and the other loadings, it is not intended to cover these here as these are well covered in other actuarial papers.

 $\frac{\left[\text{ILF}\left\{\min\left\langle Excess(P) + Limit(P), Excess(P) + Excess(R) + Limit(R)\right\rangle\right\} - \text{ILF}\left\{\min\left\langle Excess(P) + Limit(P), Excess(P) + Excess(R)\right\rangle\right]\right]}{\left\{\text{ILF}\left\{\min\left\langle Excess(P) + Limit(P), Excess(P) + Excess(R)\right\rangle\right\}\right]} \times POLPREM$

(4.1)

Where:-

Variables

Deductible (P) = Policy Deductible
 Limit (P) = Policy Limit

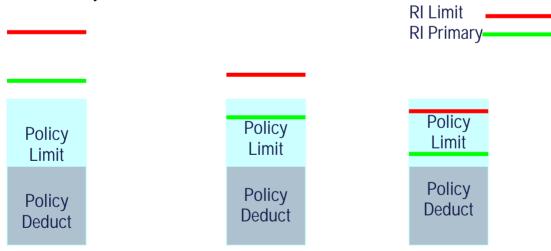
Deductible (R) = Reinsurance Deductible
 Limit (R) = Reinsurance Limit

Functions

ILF(x) = Increased Limit Factor at limit x

Min(x, y) = the minimum of x and y

This can easily be visualised as shown below:-



These calculations are performed at each risk banding within a standard "risk profile" showing information such as

- Policy count
- Premium
- Policy Limit
- Policy Deductible

The information is usually provided in bandings say \$1m, \$2m etc. Other aggregated information such as total policy limit is sometime also provided within these bands where the band contains all limits up to and including X, but not all policies will be at the maximum, so extra information is helpful in assessing the true risk and hence the reinsurance pricing. The ability to price each band naturally leads us to ask about the underlying frequency and severity distributions, this information we can extract using the information contained within the ILFs.

Other variations on the formula are required where there is coinsurance of policy limits or for Pricing Retro treaties on ventilated programs; this is left to the interested reader.

6. Business Implications

ILF curves were discussed in some detail in Miccolis, and they are extensively used in the United States, ISO calculates many ILFs and in many cases they are relied upon to In other countries (including Australia), insurance company underwriters may have a very good feel for the appropriate level of ILF. These may have been taken from elsewhere, developed over time, or be based upon what the market can bear in terms of pricing. We must not forget the commercial reality of buying higher limits, that there may be some impact of the purchaser wanting to buy and an unwilling seller that can be used to leverage up the price beyond what pure theory would suggest. The theoretical ILFs can allow for risk volatility, but it is not the purpose of this paper to consider these, and this can be left as an area for further work. Once we have a set of ILF curves, we can use the information in them for a variety of purposes, such as pricing reinsurance layers, estimating the number of losses above a threshold you may expect (this can be helpful for reserving purposes and pricing purposes). They can also be used to estimate the volatility of this amount if they are moved from a deterministic space to stochastic, and this paper covers some of the elementary mathematics required for this transition.

When setting ILFs, due consideration should be given to the marketability of the coverage, for example when there is a desire to buy higher layers and this can lead to them being priced above the economic fair value, driven by demand.

Using the information from the ILF curve, can be very useful. Especially when the ILFs have developed over time as they may consider losses beyond the normal past data re-valued starting point of many pricing exercises. By comparing the calculated results with the past (re-valued and projected) data we can see how much margin there may be in the pricing ILF factors to allow for systemic risk or the risk of black swan events (events with a small probability but with a massive impact). If we can price reinsurance layers with stochastic ILF factors based upon our assumption, it can help to allocate capital more efficiently, and this allow better strategic planning, and managing the insurance/reinsurance cycle.

7. Transformations required to Simulate Large Losses

So far, we have been able to calculate expected losses and recoveries of basic reinsurance analytically, but for more complicated scenarios, it will be desirable to sample losses, at least the large losses above a threshold y. This is used for example when pricing aggregate deductibles. Let us assume that we consider a book with losses of severity X described by the increased limit factors curve ILF. Furthermore assume that all losses are limited by a limit L, (the results can easily be extended to also include a deductible). When given the total loss E(S) of the book (for example when premium income and loss ratio can be estimated), we now describe how large losses above the threshold Y can be simulated.

As a first step, we calculate, based on some earlier results (rearrange equation (3.2) i.e. the definition of the ILF), the expected loss size of the limited severity

$$E[\min(X,L)] = E[\min(X,y)] \times ILF(L)$$
(7.1)

The expected number of losses is then given by (using equation (3.1)).

$$E(N) = E(S)/E[\min(X, L)]$$

$$= E(S)/(E[\min(X, y)] \times ILF(L))$$
(7.2)

We note that this describes losses of all sizes and hence need to be conditioned on the event $\{X > y\}$ which leads to number of losses \tilde{N} above the threshold with mean (using equation (3.5))

$$E(\tilde{N}) = E(N) \times P\{X > y\}$$

$$= E(N) \times E[\min(X, y)] \times ILF'(y)$$

$$= E(S) \times \frac{ILF'(y)}{ILF(L)}$$
(7.3)

When N has a Poisson distribution then this is the case for \tilde{N} .

Finally, we can calculate the distribution for the large losses

$$P(\min(X, L) > x | X > y) = \frac{P\{\min(X, L) > x\}}{P\{X > y\}} = ILF'(x)/ILF'(y)$$
 (7.4)

when $x \le L$ and this will be 0 for x > L.

8. Areas for Further Work

- § Consider using the APRA claims information to enable Australia ILF factors to be calculated
- § Consider approaches to the development of large losses for social inflation, IBNER and severity fitting, and measuring the impact on the ILF curves by size of loss.

9. Conclusion

We have derived formula for calculating the frequency and severity of losses above a threshold, thus we can derive suitable assumptions for stochastic modelling from ILF information. The above framework can simulate the expected losses and volatility due from a liability portfolio, using key pricing assumptions such as loss ratio, ILF curve and limit profile. The amounts are likely to be well understood by the cedant and from a reinsurer/reinsurance broker perspective. We can use this information to extract more data around uncertainty and probability that at first glance appears possible. This can lead to optimised reinsurance purchasing and improved understanding of large losses for liability classes. This can also be used to compare the actual and expected number of large losses and may be useful in a pricing exercise on scheme business with a limited amount of history from which to derive robust large loss assumptions. It may also have applications in reserving, again where the past large loss history is considered to be out of date and not appropriate for the more recent years, as is nearly always the case if the book has been re-underwritten.

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